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Mathematical model of shaping toothed products using volumetric tool with one motion parameter

A. A. Silich, E. M. Yusupova
Tyumen Higher Military Engineering Command School (Tyumen, Russian Federation)

Introduction. The development of a mathematical model of one-parameter shaping of a toothed product is considered. As an example, shaping of the side surface of the teeth of the Novikov gearing is studied; the mode and magnitude of the change in the shaping error heightwise the wheel tooth are shown. The work objective was to develop a mathematical model of the surface of the product teeth as a generating surface envelope of the tool. A computational and experimental study is carried out. The mathematical models obtained can be used in devices with copiers when shaping the side surface of the teeth of the Novikov gear. As an example, we consider the deviation behavior of the teeth profile of the Novikov gear with the original profile of DLZ 0.7-0.15

Materials and Methods. When building the model and studying its characteristics, the mathematical tools of the gearing theory, calculation procedure for cylindrical gears (A.A. Silich's author development) were used. The paper proposes new mathematical models of the equations of the lateral surface of the gear teeth formed with a tool whose axial profile coincides with the original one. In the model under consideration, the tool moves along the axis of the product while the latter rotates on its axis. In the course of the study, numerical modeling was carried out to determine the error value in shaping the product profile using the tool.

Results. New mathematical models and software have been developed for numerical simulation of the shaping of a toothed product using a tool with one independent motion parameter. An algorithm has also been developed to determine the deviation error of the real profile from the nominal one for the tooth of the Novikov gear. Solutions to accurately reproduce the tooth profile are provided.

Discussion and Conclusions. The parametric method of analytical description of the surface used in the work simplifies the calculation of the cutting tool displacements in the problems of numerical control. Solving the problem of synthesizing the technology of workpiece surface treatment on metal-cutting machines provides the development of a description of the entire shaping process and requires the representation of the workpiece surface in the form of a mathematical model. The results obtained can be used under creating finishing methods for processing teeth when improving the quality of gear wheels and gear drives, as well as the production efficiency.

Keywords: toothed products, mathematical model, one-parameter shaping, error of shaping.

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Introduction. The process of shaping the tooth flank of the Novikov gear wheel is considered using the following algorithm:

— generating surface of the tool is described in the coordinate system of this tool;

— real surface of the wheel teeth is formed as an envelope of the generating surface of the tool when the latter

moves relative to the wheel with independent parameters;

— numerical studies on the deviation of the real surface from the theoretical one and the analysis of the results obtained are carried out.

In accordance with GOST 13 755–81 for cutting tools, the standard establishes a normative number of modules and certain relationships between the dimensions of tooth components. These ratios are determined for gears by the parameters of the original rack through the parameters of its normal section — the original profile. A grinding wheel is considered as a tool in the work.

The relative motion of the gear links or the tool and the workpiece during machining can be described by one or two independent parameters. The surface of the teeth of the first link is an envelope of a one-parameter or two-parameter family of surfaces of the second link. Very often under processing, the surface of product teeth is formed as an envelope of the generating surface of the tool when the latter moves relative to the product with one independent parameter. In most cases, the angle of rotation of the product is chosen as such a parameter.

The theoretical surface of the wheel teeth in the process of its finishing with an abrasive tool is formed as an envelope of the generating surface of the tool when the latter moves relative to the wheel with independent parameters. According to GOST 16 530–83, the theoretical surface is each of two surfaces (lateral surfaces of two teeth) providing a given gear ratio during their interaction.

Having regard to the above, the task was set to determine the equations of the side surface of the product teeth. The surfaces are created with a tool whose axial profile matches the original profile. In this case, the tool moves along the axis of the product, and the wheels perform conformal eigenrotation. Since the rotation of the product and the movement of the tool are interconnected through an analytical relationship, the angle of product rotation is chosen as an independent parameter for the shaping of a toothed product $\varphi_к$.

Materials and Methods. To obtain the equation of the nominal generating surface of the tool, a coordinate system rigidly connected with the rack was used. The normal rack section is taken as the reference profile. In Fig. 1, a generalized reference profile taken from the research papers [1, 2] is exemplified. The profile is made up of circular segments that can smoothly mate with each other or intersect depending on the type of the reference profile.

In Fig. 1, the numbers in circles show the numbers of the sections (i=1,2,3,…7). The boundaries of the sections are marked with large dots. In addition, in Fig. 1 the following designations are accepted: S_i $(X_i;Y_i;Z_i)$ coordinate systems associated with the i-th section of the reference profile. The origin of such a system is aligned with the center of the section circle, and the direction of the axes coincides with the direction of the axes of the rack S_p $(X_p;Y_p;Z_p)$.

The solution to the problem is divided into two stages. At the first stage, a mathematical model of a volumetric tool in the form of a body of revolution (e.g., a disk grinding wheel or a disk cutter) is built. At the second stage, a toothed product with one independent motion parameter is formed using the tool.

Tool mathematical model

We obtain a theoretical generating surface through binding the surface of the tool to the coordinate system S_u $(X_u;Y_u;Z_u)$ directing the Z_u axis along the tool axis of rotation. We position the X_u axis so that it passes through the calculated point, which we take as the point of contact between the tool and the workpiece at the initial moment of time. The Y_u axis is directed so that all axes make up a right-hand Cartesian coordinate system. The reference profile is associated with its own coordinate system $S_p(X_p;Y_p)$, whose axes are located so that the Z_p axis coincides with the pitch line of the reference profile, and the X_p axis is perpendicular to it and directed towards the tool axis.

To shape the generating surface of the tool, devices are used that work with a copier, or devices that reproduce the trajectory of a dressing diamond or a sharpening tool using a NC system. For any of these cases, a mathematical description of the tool generating surface will be of the same type[1, 2] [3].

[1] Krivosheikin AV, Nurmukhamedov LKh, Perelygin SV. Mathematical modeling in instrumentation systems. St.Petersburg, 2019. 108 p. (In Russ.)
[2] Zyuz'kov VM. Mathematical logic and theory of algorithms. Tomsk, 2015. 236 p. (In Russ.)

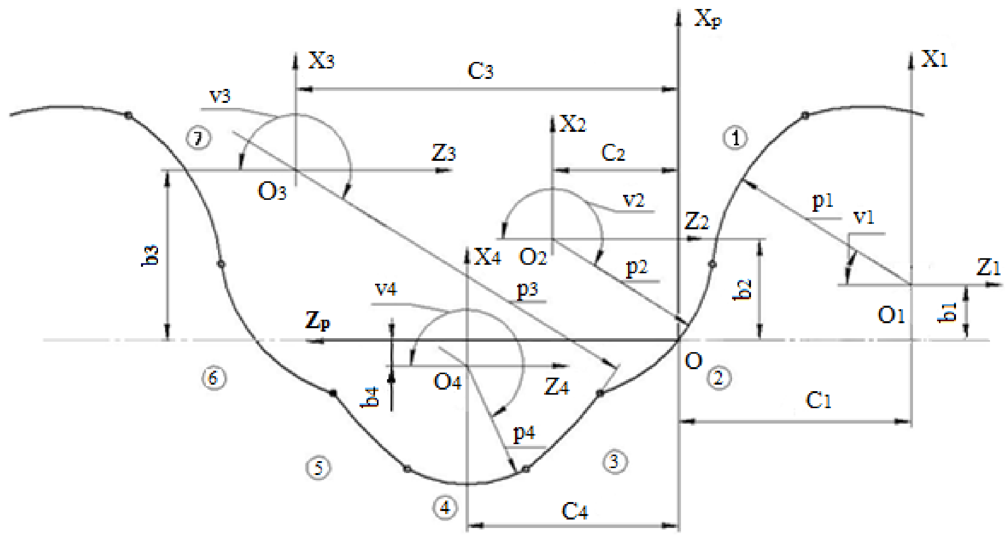

Fig. 1. Reference profile DLZ-0.7-0.15

Fig. 2 shows the relative position of the tool (1), the reference profile (2), the copier (3), the correct device diagram and the adopted coordinate systems associated with the copier and the tool.

The mathematical model of the tool generating surface will be considered as a trace of the copier contour motion when it rotates around the Z_u axis with the angular parameter of movement φ_u.

In the S_p coordinate system, the copier profile equation including sections 1, 2 and 3 can be written as follows:

$$\left.\begin{matrix} X_p = \rho_i \cdot \sin \upsilon_i + b_i; \\ Z_p = \rho_i \cdot \cos \upsilon_i + c_i; \end{matrix}\right\}, \tag{1}$$

where the designation of the values are taken from [4–6]: ρ_i is the radius of curvature of the i-th section of the normal profile of the rack tooth; υ_i is the curvilinear coordinate of the rack generating surface, whose origin goes clockwise from the Z_p axis; c_i is the applicate of the position of the center of curvature of the corresponding section in the S_p system; b_i is the abscissa of the center of curvature of the corresponding section in the S_p coordinate system.

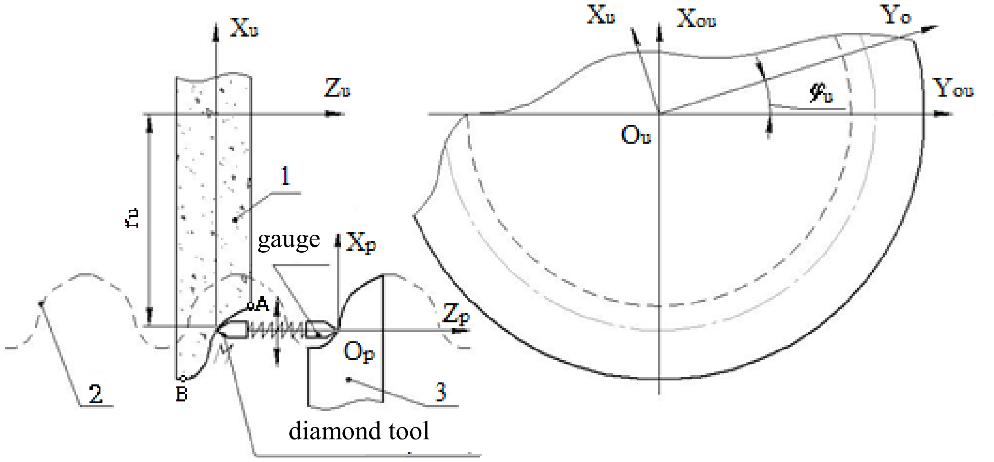

Fig. 2. Scheme of tool shaping with the adopted coordinate system

In addition, Fig. 2 indicates the calculated radius of the tool r_u.

Using the selected coordinate systems, the introduced designations and the method of shaping surfaces[3], we obtain the equation for the generating surface of the tool in projection on the coordinate axis:

$$\widetilde{r_и} = \begin{Bmatrix} X_{ui} = \cos \varphi_{ui} \cdot (\rho_i \cdot \sin \upsilon_i + b_i - r_u); \\ Y_{ui} = -\sin \varphi_{ui} \cdot (\rho_i \cdot \sin \upsilon_i + b_i - r_u); \\ Z_{ui} = -\rho_i \cdot \cos \upsilon_i - c_i. \end{Bmatrix} \tag{2}$$

To obtain a mathematical model of the product, the equation of the normal to the generating surface of the tool

[3] Rannev GG, Tarasenko AP. Intelligent measuring instruments. Moscow, 2016. 280 p. (In Russ.)

is required. Therefore, using the well-known technique [3] and omitting the intermediate transformation, we write the equations of the unit normal vector to the generating surface in the following form:

$$\left.\begin{aligned} e_{xui} &= -\cos\varphi_u \cdot \sin\upsilon_i; \\ e_{yui} &= \sin\varphi_u \cdot \sin\upsilon_i\,; \\ e_{zui} &= \cos\upsilon_i. \end{aligned}\right\} \quad (3)$$

Product mathematical model

The geometrical-kinematic diagram of the shaping of the product tooth lateral surface using a tool with the appropriate coordinate systems is shown in Fig. 3. Sections of the tool generating surface are specified in the S_u (X_u; Y_u; Z_u) coordinate system rigidly connected with the tool. The coordinate system $S_к$ ($X_к$;$Y_к$;$Z_к$) is rigidly connected with the workpiece (e.g., a gear wheel). Fixed auxiliary coordinate systems of the wheel S_{ou} (X_{ou};Y_{ou};Z_{ou}) and the tool $S_{ок}$ ($X_{ок}$;$Y_{ок}$;$Z_{ок}$) associated with the rack.

As an independent parameter of the relative motion under the product processing, we take the value $\varphi_к$ numerically equal to the angle of rotation of the wheel around the $Z_к$ axis. The tool motion along the workpiece axis is denoted by S_o. The angle of inclination of the tooth line of the product is $\beta_к$. The axle spacing is $a=r_u+r_k+X_k$.

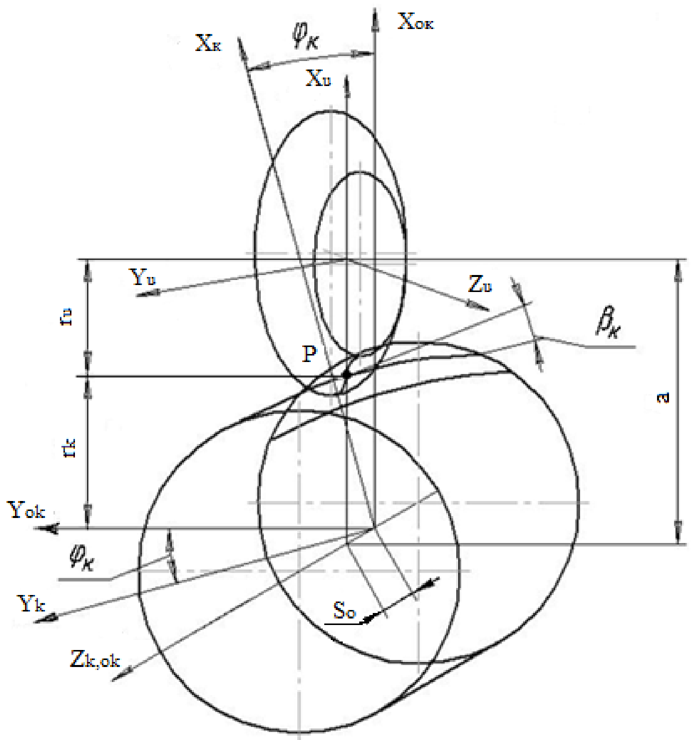

Fig. 3. Geometrical-kinematic diagram of the product tooth profile shaping by the tool

The real product tooth surface as the envelope of the one-parameter family of the tool generating surface is determined from the following equations [3]:

$$\left.\begin{aligned} \tilde{r}_к &= \tilde{A}_{кui} \cdot \tilde{r}_u && \text{а)} \\ \bar{e}_{ui} \cdot \bar{V}_{ui} &= 0 && \text{б)} \end{aligned}\right\}, \quad (4)$$

where $\tilde{r}_u$ is the column matrix composed of the projections of the tool generating surface recorded in the S_u coordinate system; $\tilde{A}_{кui}$ is the transition matrix from the S_u coordinate system to the $S_к$ coordinate system; $\bar{V}_{ui}$ is the prototype of the relative velocity vector by parameter φ_u; $\bar{e}_{ui}$ is the unit normal vector to the tool generating surface, whose projections in the X_u, Y_u, Z_u coordinate axes are represented by the equations (3).

We find the transition matrix $\tilde{A}_{кui}$ using the technique described in [3, 7]. Omitting intermediate transformations, we present this matrix in the following form:

$$\tilde{A}_{кui} = \begin{pmatrix} \cos\varphi_к & \sin\beta_к \cdot \sin\varphi_к & -\cos\beta_к \cdot \sin\varphi_к & a\cdot\cos\varphi_{ui} \\ -\sin\varphi_к & \sin\beta_к \cdot \cos\varphi_к & -\cos\beta_к \cdot \cos\varphi_к & -a\cdot\sin\varphi_{ui} \\ 0 & \cos\beta_к & 1 & -S_o \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (5)$$

$$S_o = \frac{\varphi_к \cdot r_к}{\tan\beta_к} = \frac{\varphi_к \cdot m \cdot z_к}{2\cdot\sin\beta_к}$$

$$a = r_к + X_к + r_u = \frac{m\cdot z_к}{2\cdot\cos\beta_к} + X_к + r_u.$$

Using the technique of one-parameter shaping of surfaces [3], adopted coordinate systems and geometrical-kinematic shaping scheme (Fig. 3), as well as omitting intermediate transformations, the equation of the real surfaces of the product teeth can be written as:

$$\left.\begin{aligned} x_{\kappa i} &= \cos\varphi_\kappa\cdot\cos\varphi_u\cdot\sin\beta_\kappa\cdot\sin\varphi_\kappa\cdot\sin\varphi_u\cdot(\rho_i\cdot\sin\upsilon_i+b_i-r_u)+\\ &\quad+\cos\beta_\kappa\cdot\sin\varphi_u\cdot(\rho_i\cdot\cos\upsilon_i+c_i)\\ y_{\kappa i} &= \sin\varphi_\kappa\cdot\cos\varphi_u\cdot\sin\varphi_u\cdot\sin\beta_\kappa\cdot\cos\varphi_\kappa\cdot(\rho_i\cdot\sin\upsilon_i+b_i-r_u)+\\ &\quad+\cos\beta_\kappa\cdot\cos\varphi_\kappa\cdot(\rho_i\cdot\cos\upsilon_i+c_i)\\ z_{\kappa i} &= -\cos\beta_\kappa\cdot\sin\varphi_u\cdot(\rho_i\cdot\sin\upsilon_i+b_i-r_u)-\\ &\quad-\sin\beta_\kappa\cdot(\rho_i\cdot\cos\upsilon_i+c_i)-\varphi_k\cdot r_k\cdot ctg\,\beta_\kappa \end{aligned}\right\} \quad (6)$$

Using the technique of obtaining the gearing equation (4 b)[4], we finally write it in the following form:

$$\begin{aligned}\cos\beta_\kappa\cdot[\sin\varphi_{ui}\cdot\sin\upsilon_i\cdot(c_i-r_k\cdot\sin\beta_\kappa+X_k\cdot\tan\beta_\kappa+r_u\cdot\tan\beta_\kappa)-\\ -\cos\upsilon_i\cdot(r_k+X_k)]=0\end{aligned}, \quad (7)$$

where X_κ — reference profile displacements.

Research Results

To determine the error value in shaping the product profile using the tool, the following algorithm was used:

1. The real and nominal surfaces of the product teeth were recorded in the same coordinate system. The equations of the lateral surface of the wheel teeth formed using the rack, were taken as the nominal surface of the product teeth[5] [3].
2. The current value of the independent parameter υ_j, was set, where j=1, 2, 3…n is the current point number on the wheel tooth profile.
3. Taking $Z_{\kappa i}=const$, the value of the second independent parameter $\varphi_{\kappa j}$ of the nominal product profile was determined.
4. A circle was drawn through the current point υ_j of the product tooth profile, its intersection with the product real tooth profile was determined.
5. The chord distance between the points of the nominal and real profile located on the same circle was taken as the shaping error.

The developed algorithm for determining the shaping error was implemented as a program in the MathCAD software environment.

As an example, Fig. 4 shows the real profile behavior in comparison with the nominal one, and the direction of reading the error Δ_{ij} of the shaping of the Novikov gear wheel tooth profile.

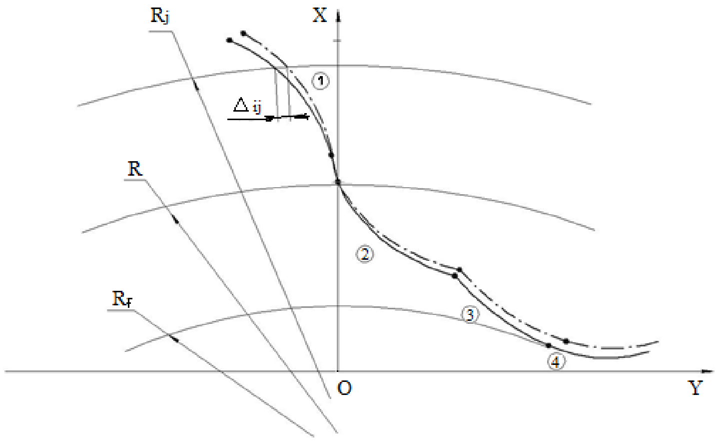

Fig. 4. Behavior of the Novikov gear wheel teeth profile:
——— — nominal profile; — · — — real profile.

Real values of the errors Δ_{ij} are shown in Fig. 5 for a Novikov gearwheel with the reference DLZ profile of 0.7–0.15 and the following geometrical parameters: gearing modulus m_n=5, number of the wheel teeth z_κ=50, angle of inclination of the tooth line β_κ=20º, displacements of the reference profile X_κ=0.

[4] Rannev GG, Tarasenko AP. Op. Cit. 280 p. (In Russ.)
[5] Zyuz'kov VM. Op. Cit. 236 p. (In Russ.)

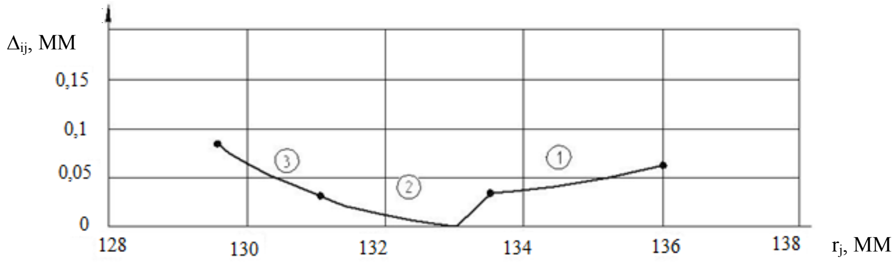

Fig. 5. Deviation of the real profile from the nominal one for the tooth of the Novikov gear wheel

As can be seen from the graph in Fig. 5, the minimum value of the deviation is in the area of the pitch radius of the wheel, and the maximum values are observed on the head and at the base of the wheel tooth. The maximum values of profile shaping depend on the type of the reference profile, the number of teeth, the angle of inclination of the tooth line, as well as on the diametrical dimensions of the tool.

Discussion and Conclusions. A mathematical model of the shaping of a toothed product using a tool with one independent motion parameter has been developed.

1. Numerical studies show that according to this processing scheme, it is impossible to theoretically accurately reproduce the product tooth profile if the generating surface of the tool is formed by the reference profile of the workpiece.

2. The maximum distortions of the profile are observed on the head and foot of the wheel tooth (at the diameter of the protrusions and depressions) and their value is the greater, the higher the reference profile, the greater the module, the smaller the number of teeth, and the greater the angle of tooth inclination.

3. For accurate reproduction of the tooth profile, it is required to correct the copier, which is used to dress and sharpen the tool.

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About the Authors:

Silich, Aleksandr A., associate professor of the Department of Military-Technical Disciplines, Tyumen Higher Military Engineering Command School (1, Tolstogo St., Tyumen, 625001, RF), Dr.Sci. (Eng.), professor, ORCID: https://orcid.org/0000-0002-1114-4726, silichaas@gmail.com

Yusupova, Elvira M., lecturer of the Natural Sciences and General Professional Disciplines Department, Tyumen Higher Military Engineering Command School (1, Tolstogo St., Tyumen, 625001, RF), ORCID: https://orcid.org/0000-0002-0463-1067, jusupovaem@tyuiu.ru

Claimed contributorship

A. A. Silich: academic advising; analysis of the research results; basic concept formulation. E. M. Yusupova: research objective and task setting; computational analysis; text preparation; formulation of conclusions.

All authors have read and approved the final manuscript.